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LETTER TO THE EDITOR

Critical dynamics of a dilute central force network with partial bond bending forces

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Abstract. Numerical results are presented for the dynamical properties of a dilute, central force, triangular network with 60° bond bending forces. The rigidity percolation threshold P_R for such a network lies between the connectivity percolation threshold and the central force rigidity percolation threshold. A LU factorisation technique is used to determine the eigenvalues of the dynamical matrix and the results combined with finite size scaling to yield values for P_R and the fracton dimensionality $\tilde{d}$ of $0.40 \leq P_R \leq 0.405$ and $1.25 \leq \tilde{d} \leq 1.3$.

During the past decade, despite an early suggestion to the contrary (de Gennes 1976), it has become clear that electrical networks constitute a different universality class from elastic structures. Indeed, recent work has shown that elastic networks themselves divide into different universality classes. Whilst a variety of models with varying degrees of complexity have been examined (Sen *et al* 1985, Garcia-Molina *et al* 1988, Odagaki 1989) much theoretical work to date has focused on dilute networks with single bond strengths, or their superelastic counterparts.

Several authors have also addressed the question of whether or not for a given dimensionality d, elastic networks with a single bond strength only, themselves divide into different universality classes. The main evidence for this division comes from numerical calculations on bond dilute central force triangular networks. If bond bending forces are included between all bonds emanating from a site, an elasticity exponent (Zabolitzky *et al* 1985, Sahimi 1986) of $f_{bb} = 3.96 \pm 0.04$ is obtained, whereas early estimates (Day *et al* 1986) of the same exponent for a lattice with central forces only, yield $f_{cen} = 1.4 \pm 0.2$. Recently however (Roux and Hansen 1988), the latter value has been questioned and a very different estimate of $f_{cen} = 3.15 \pm 0.5$ has been obtained. If the latter estimate is correct, then the question of whether or not dilute two-dimensional, percolating elastic networks, with a single bond strength only, divide into more than one universality class, remains open.

In this letter we address this question by examining the dynamical properties of a new model, which falls between the two single bond strength models described above. The model comprises a central force triangular network with 60° bond bending forces only. This is a new model for elasticity percolation. To our knowledge, not even the rigidity percolation threshold of this structure is known. In what follows, we demonstrate that the model possesses dynamical exponents which differ from the full bond bending

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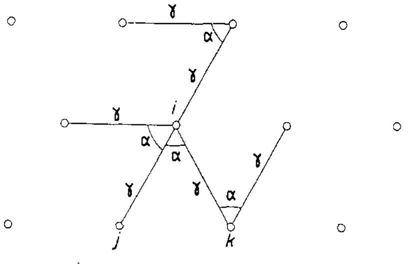

Figure 1. This figure shows a typical configuration of central force bonds γ and angular bonds α in the vicinity of an arbitrary site i. As an example, the bond α associated with the angle θ_{ijk} is present because both central force springs connecting i to j and i to k are present. If either of these springs is removed, then the bond bending term associated with θ_{ijk} is set to zero.

universality class and are in fact rather close to the values obtained using early estimates (Day *et al* 1986) for f_{cen}. The potential energy for the network, obtained by summing over all central force bonds γg_{ij} connecting nearest neighbours $\langle ij\rangle$ and all 60° bonds $\alpha g_{ij} g_{ik}$ associated with a triplet of nearest neighbours $\langle ijk\rangle$ (see figure 1), where $g_{ij} = 1$ with probability P and $g_{ij} = 0$ otherwise, is given by

$$H = \tfrac{1}{2}\gamma \sum_{\langle ij\rangle} [(\boldsymbol{U}_i - \boldsymbol{U}_j)\cdot \hat{\boldsymbol{r}}_{ij}]^2 g_{ij} + \tfrac{1}{2}\alpha \sum_{\langle ijk\rangle} (\delta\theta_{ijk})^2 g_{ij} g_{ik}. \tag{1}$$

In this expression $\hat{\boldsymbol{r}}_{ij}$ is a unit vector along bond ij, $\boldsymbol{U}_i$ is the displacement of site i and $\delta\theta_{ijk}$ is the deviation from 60° of the angle between neighbours j and k of i. In what follows, we set $\alpha = \gamma = 1$.

For an infinitely large system, if ξ is the correlation length of the network near its rigidity percolation threshold P_{R}, then the long wavelength velocity of sound C varies as $C^2 \sim \xi^{-\theta}$, where θ is related to the mass, correlation length and elasticity exponents β, ν and f by the expression (Alexander and Orbach 1982)

$$\theta = f/\nu - \beta/\nu. \tag{2}$$

For a frequency ω below the crossover frequency $\omega_\xi \sim C/\xi \sim \xi^{-(2+\theta)/2}$ one expects a Debye-like integrated density of states per unit volume of the form

$$N(\omega) \sim \omega^d/C^d \qquad (\omega \ll \omega_\xi). \tag{3}$$

More generally, invoking a single parameter scaling hypothesis, one expects for an infinitely large system

$$N(\omega) = (\omega^d/C^d)g(\omega/\omega_\xi) = \xi^{-d} f(\omega/\omega_\xi) \tag{4}$$

where $f(\Omega)$ is a dimensionless function of the dimensionless frequency $\Omega = \omega/\omega_\xi$. The assumption $f(\Omega) \rightarrow \Omega^{\tilde{d}}$ for large Ω and that for large Ω, $N(\omega)$ is independent of ξ, yields the result $\tilde{d} = [2d/(2+\theta)]$. The resulting integrated density of states per unit volume for these higher frequency fractons, is of the form $N(\omega) \sim \omega^{\tilde{d}}$. By analogy with equation (3), $\tilde{d}$ is the effective dimensionality which characterises the frequency dependence of the fracton spectral density, in much the same way that the Hausdorff dimension $\bar{d} = d - \beta/\nu$ (Stinchcombe 1985), characterises the mass density of the infinite percolating cluster.

For a system of finite size L, the system size provides a second length scale, which must be included in the scaling relation for $N(\omega)$. In this case equation (4) is replaced by

$$N(\omega) = \xi^{-d} g(\omega/\omega_\xi, \xi/L) = L^{-d} f(\omega/\omega_L, \xi/L) \tag{5}$$

where f and g are new scaling functions, $\omega_L = L^{(2+\theta)/2}$ and ξ is a function of both P and

L. For $\xi \ll L$, one expects ξ to be independent of the system size L and of the form (Stinchcombe 1985)

$$\xi \sim |P - P_R|^{-\nu}. \tag{6}$$

However, as P approaches P_R, since ξ cannot exceed the system size, one expects ξ/L to approach a value of order unity. Under the assumption that for large L, at $P = P_R$ this value approaches a limit α, equation (5) can be used to determine both the rigidity percolation threshold P_R and the parameter θ, by first computing $N(\omega)$ versus ω for various values of L. For a given choice of P and θ, plots of $N(\omega)L^d$ versus $\omega L^{-(2+\theta)/2}$ are constructed. If $P = P_R$ and θ is correctly chosen, equation (4) predicts that for large L, the plots will fall onto a universal curve $f(\Omega, \alpha)$. Thus by performing a search of the P, θ space, the correct values of θ and P_R are simultaneously determined.

The parameter θ is the elastic equivalent of the anomalous diffusion exponent which arises in random walker simulations for the dynamical properties of percolating networks (Alexander and Orbach 1982). To our knowledge, such simulations have not been performed on models of elasticity percolation. Nevertheless, using the values for f_{cen} and f_{bb} quoted above, along with the corresponding values for β and ν, estimates for θ_{cen} and θ_{bb} can be obtained from equation (2). For the full bond bending model $\beta = 5/36$ and $\nu = 4/3$, which yields $\theta_{bb} = 2.87 \pm 0.04$ and hence $\tilde{d}_{bb} = 0.82 \pm 0.01$. For central forces only $\nu = 1.05 \pm 0.15$ (Day *et al* 1986). In this case, we are aware of no quoted value for β in the literature. However, if all dangling bonds are removed from the spanning cluster, the mass exponent for the resulting percolating backbone has been estimated (Day *et al* 1986) at $0.06^{+0.11}_{-0.06}$, which provides an upper limit for β. (The lower limit for β is of course zero.) Hence if $f_{cen} = 1.4 \pm 0.2$, then $\theta_{cen} = 1.3 \pm 0.5$ and $\tilde{d}_{cen} = 1.2 \pm 0.2$. On the other hand, Roux and Hansen's value for f_{cen}/ν yields $\theta_{cen} = 2.9 \pm 0.5$ and $\tilde{d}_{cen} = 0.8 \pm 0.1$. In what follows, we compare these results with values of θ and d obtained for the model defined by equation (1).

To compute $N(\omega)$, for the partial bond bending model of equation (1) the corresponding dynamical matrix was LU factorised using standard APMATH 64 sparse matrix routines on a FPS 264. Results for a triangular network of L^2 sites with periodic boundary conditions were obtained for $L = 12, 24, 36$ and for values of P ranging from $P = 0.39$ to $P = 0.415$ in steps of 0.05. For $P = 0.40$, figure 2 shows the density of squared frequencies $\mathrm{d}N(\omega)/\mathrm{d}(\omega^2)$ for the largest of these system sizes. The higher frequency region ($\omega \gtrsim 0.5$) of this curve is not expected to obey scaling, since its behaviour is dominated by the Debye cut-off of the lattice. On the other hand, values of P and θ can be found, for which the fracton region ($\omega < 0.5$) clearly exhibits scaling. A limited selection of our results for the scaled, integrated density of states is shown in figure 3. For each system size, the results were averaged over 108 configurations, yielding error bars which are smaller than the symbol sizes used in the figures. For $P = 0.40$, $\theta = 1.05$ and $P = 0.405$, $\theta = 1.2$ the results appear to scale equally well. Outside this range, the results do not fall on a single universal curve. The corresponding values for $\tilde{d} = 2d/(2 + \theta)$ are $\tilde{d} = 1.3$ at $P_R = 0.40$ and $\tilde{d} = 1.25$ at $P_R = 0.405$.

These results suggest that the 60° bond bending model analysed in this letter does not fall into the same universality class as a full bond bending model. Hence two-dimensional dilute elastic networks, with a single strength bond only, divide into at least two universality classes. It is interesting to note that to within the quoted error bars, the exponent θ coincides with early estimates (Lemieux *et al* 1985) of θ_{cen}, so there is a possibility that the partial bond bending model falls into the same class as central force networks. To confirm the possibility, or otherwise, there is clearly a need for an accurate

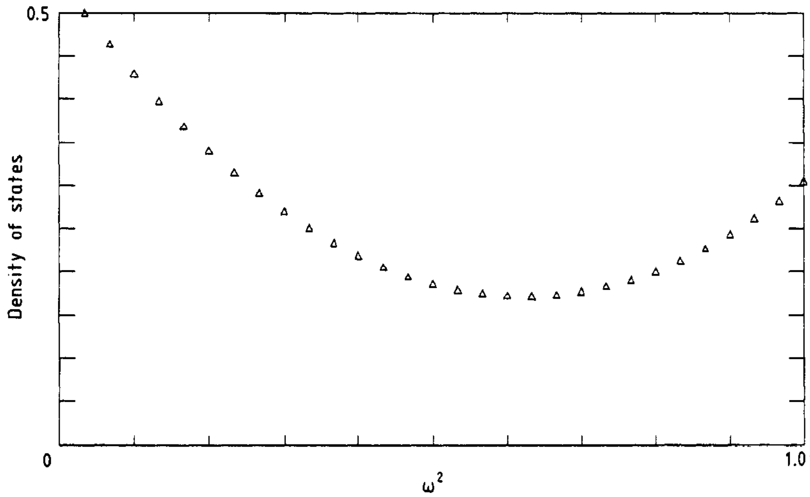

Figure 2. The density of squared frequencies $dN(\omega)/d(\omega^2)$ for $P = 0.4$ and $L = 36$. The fracton region, for $\omega^2 < 0.5$ exhibits a power law behaviour of the form $\omega^{\bar{d}-2}$.

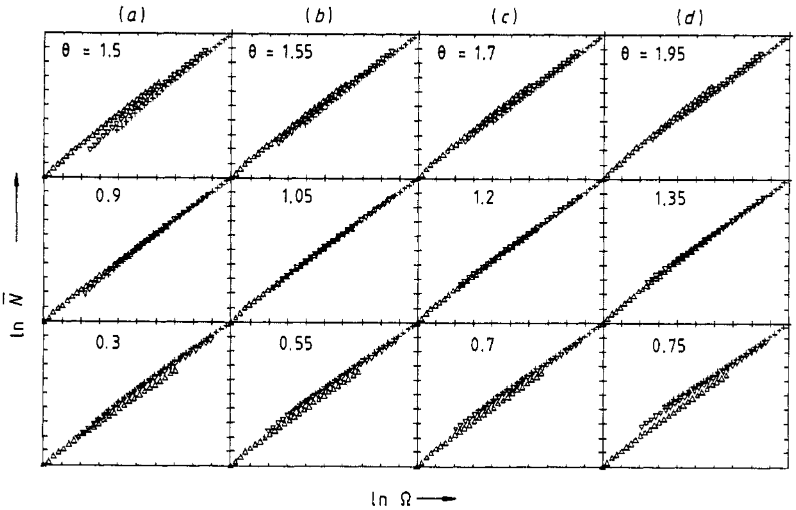

Figure 3. Typical scaling plots of $\bar{N}(\omega) = N(\omega)L^d$ against $\Omega = \omega L^{-(2+\theta)/2}$ for system sizes $L = 12\,(\triangle), 24\,(\nabla)$ and $36\,(+)$. The values of P are (*a*) 0.39, (*b*) 0.40, (*c*) 0.405 and (*d*) 0.415. Values of ω, for each plot, are in the range $10^{-3} \leqslant \omega^2 \leqslant 0.5$.

numerical simulation which is capable of resolving the present discrepancy between quoted values of the central force exponents. We have recently initiated such a calculation and hope to report the results in the near future.

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